

On a Class of Operator Ranges in Dual Spaces

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Abstract

In this lecture, we provide a characterization of the adjoint operator ranges of the dual of a Banach space E , that is, the subspaces of E^* that are the range of the adjoint of a bounded linear operator between E and another Banach space. We also show that the space E^* contains an adjoint proper dense operator range if, and only if, there exists a weak*-closed subspace Z of E^* such that E^*/Z is infinite-dimensional and separable (which is equivalent to the existence of an infinite-dimensional Asplund subspace of E). As an application, we derive a characterization of Asplund-saturated Banach spaces. Finally, we establish some results concerning the existence of quasicomplementary subspaces with a prescribed behavior with respect to adjoint proper dense operator ranges in dual spaces.